**Rotational Inertia[[1]](#footnote-1)\***

**Background**

As you’ve already learned, linear motion and rotational motion are completely analogous. There are measurements of position,  and , velocity,  and , and acceleration  and . In this lab, we will also consider how inertia is represented.

In linear motion, mass is a measure of the inertia of an object, and is measured in kilograms. In rotational motion, we need to consider the *rotational inertia* of an object instead. Rotational inertia takes into account not only the total mass of an object, but how it is distributed. Rotational inertia is represented as  and has units of  The most general form of the rotational inertia has an integral calculus representation - however, your book has a table with the specific equation for rotational inertia for various shapes. The two shapes that we are using today are a ring and a disk, and are written as

Another important concept is forces, and you know that forces cause motion. Similarly, a *torque* is a special kind of force that causes rotational motion. The most important part of a torque is that it acts as a lever arm. Think of using a wrench to loosen a bolt: the further out the handle you apply a force, the easier it is to get the bolt to spin! The equation for torque takes into account the force that you exert, the lever arm (distance from the pivot) and the angle that the force is applied. In equation form,



where  is the lever arm and the units of torque are 

Now, we can put all of this together to write Newton’s 2nd Law for Rotations. You already know Newton’s 2nd Law as , so using all of the analogous variables, we can write

.

(Remember the sign conventions for rotation: counterclockwise is positive.)

**Goal:** To find the rotational inertia of a disk and ring both theoretically and experimentally using Newton’s 2nd Law for Rotations, and compare the results.

**Part I - Theoretical Values of Rotational Inertia**

To calculate the theoretical values of rotational inertia, you’ll need the physical variables for the disk and the ring.

* Measure the mass of the disk and record it here:

 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ kg

* Measure the radius of the disk and record it here:

 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ m

* Measure the mass of the ring and record it here:

 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ kg

* Measure the inside radius of the ring and record it here:

 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ m

* Measure the outside radius of the ring and record it here:

 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ m

* Using the equations above, calculate the rotational inertia of the disk and the ring:

 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ kgm2

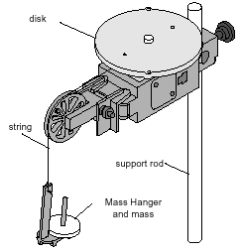
 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ kgm2

**Part II - Setting up Newton’s 2nd Law for rotations**

As with all problem involving forces and torques, a first logical step is to set up free body diagrams. In this experiment, there is a falling mass and a spinning disk, as shown in the image on the next page. Notice that the string is wrapped around the pulley, not the disk itself. So, you will need the radius of the “drum,” or the part of the pulley that the string is wrapped around. Use one of the top two drums (don’t use the largest one).

* Measure the outside radius of the pulley and record it here:

 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ m



* Draw a free body diagram for the falling mass and also for the rotating drum. Be sure to indicate the forces, lever arms, and the direction of linear and/or rotational acceleration.
* Remember that the goal is to solve for the rotational inertia, and that the rotary motion sensor will allow you to find the rotational acceleration of the system. So, the next goal will be to write an equation for  in terms of 
  + Write Newton’s 2nd Law for the falling mass and Newton’s 2nd Law for Rotations for the spinning disk. The first line for each has been started for you.

* + As you can see, both of these equations have the tension in them! Since we cannot measure tension (and don’t need to solve for that variable) it will be easiest to eliminate it from our equation. To do that, solve for the tension in each equation and set them equal.
  + Now use the relationship between the linear and rotational accelerations, , to change all of the linear accelerations to angular accelerations. Solve for the rotational inertia.

When you’ve finished, you should get an equation that looks something like this: . Or, if you’ve factored, it may look like . Remember that this is the radius of the drum from the rotational motion sensor, and the is the total mass hanging from the pulley (refer to the photo above).

**Part III - Experimentally Determining Rotational Inertia**

Now that the theory is complete, it is time to conduct experiments that will allow for the experimental determination of the rotational inertia.

* Tie a string to the spool of the rotary motion sensor (be sure to use the drum that you measured above) and connect the other end to a mass hanger. Attach the silver disk to the top of the rotary motion sensor using the small screw, and mount the entire thing on the stand. Drape the mass over a pulley as shown in the diagram above.
* Set up the Capstone program and connect a *Rotary Motion Sensor* to the 850 Interface. Create a graph of angular velocity versus time (this will allow you to solve for the rotational acceleration of the system).
* Add mass to the mass hanger and record the total value. A mass of 25-40g works well.

= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ kg

* Wind the string all the way up. Release the system and then immediately press the ***Record*** button. Press ***Stop*** before the falling mass reaches the bottom of motion. Use the *Fit* functions to find the rotational acceleration of the motion from the graph that you’ve created. Repeat this at least three times - complete the table below to find the average acceleration and the rotational inertia.

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | Average | Rotational Inertia |
| Trial #1 |  |  |  |
| Trial #2 |  |
| Trial #3 |  |
| Trial #4 |  |

* Mount the ring on top of the disk such that the pins match with the holes. Repeat the experiments and record your data. Remember that rotational inertia is simply additive, so .

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | Average | Rotational Inertia |
| Trial #1 |  |  |  |
| Trial #2 |  |
| Trial #3 |  |
| Trial #4 |  |

**Summary Questions**

1. How do the theoretical and experimental values of rotational inertia compare? Find the percent error in the experimental measurements.
2. What factors do you think contributed to errors in this experiment?

**Conservation of Angular Momentum[[2]](#footnote-2)\***

**Background**

The list of analogous variables between linear and rotational motion continues through momentum. In linear motion, you learned that if a system has no external forces, the total momentum of the system had to be conserved. That is,

 where 

In rotational motion, we can similarly define the angular momentum as



and know that if there are no external torques on a system, then the total angular momentum of the system must be conserved. This is known as the *Conservation of Angular Momentum*.

**Goal:** To examine the Angular Momentum of a system and explore the Conservation law.

**Part I - Theoretical Components**

In this lab, you’ll be using the silver disk and the black ring that you used in the Rotational Inertia lab. Refer to that lab so that you don’t have to recalculate all of the quantities!

* Using the previous data, record the theoretical values for rotational inertia below:

 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ kgm2

 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ kgm2

* In this lab, you’re going to start the disk spinning and then drop the ring down on to it. Use that information to write the conservation of angular momentum equation for the ring and the disk, both initially and finally. The first line is started for you:



**Part II - Angular Momentum Experiment**

As described above, this lab will examine the angular momentum before and after a collision of two spinning objects.

* Connect a *Rotary Motion Sensor* to the Capstone interface. Mount the sensor on a support base and rod, and attach the silver disk using a small screw - make sure that the disk is level with the table. This lab works best if the sensor and disk are relatively close to the table, low enough for you to look at the top of the disk.
* Before you collect data, practice the collision a few times. First, start the silver disk spinning. Then, drop the black ring on to the disk. Be sure that the pins are up so that it lands flat. Also, you can only get good data if you drop the ring in the center of the disk, which is easiest to do if you drop the disk from a short distance above the ring. You might want to add a bit of tape to the bottom of the ring to prevent it from slipping when you drop it onto the disk.
* Now that you’ve practiced, it’s time to collect data. Create a graph of *angular velocity versus time.* Give the disk a spin and then press ***Record.*** Drop the ring horizontallyin the center of the disk. Press the ***Stop*** button.
* If your ring does not drop and stay centered, take your data again.
  + Use the *Delta Tool* to find the angular velocity ***just before*** the collision. Then, find the angular velocity ***just after*** the collision. Record your data below and complete the table.
  + Repeat the experiment to collect three sets of data. Remember, the equation for the percent difference is .

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |
| Trial #1 |  |  |  |  |  |
| Trial #2 |  |  |  |  |  |
| Trial #3 |  |  |  |  |  |

**Summary Question**

Do your experimental results for the angular momentum agree with the theory? Discuss what factors in the experiment may have introduced uncertainty.

1. \* Adapted from a *Science Workshop 500* lab by PASCO, Inc.

   Image credits: PASCO Incorporated, Roseville CA [↑](#footnote-ref-1)
2. \* Adapted from a *Science Workshop 500* lab by PASCO, Inc.

   Image credits: PASCO Incorporated, Roseville CA [↑](#footnote-ref-2)